SAMYRAD 2026

SMART ALGORITHMS & MODELS: YOUR REACH TO ADVANCED DIALOGUES · 5–6 OCTOBER 2026 · SEVILLE, SPAIN
SAMYRAD OPEN-ACCESS PROCEEDINGS | PUBLISHER: INTELLIGENT NUDGE

PAPER • OPEN ACCESS

Enhancing geometry exemplification through integration of digital technology tools in South African higher education mathematics classrooms

Annie M. Kgosi and Anthony A. Essien
University of the Witwatersrand, South Africa

Article DOI: 10.68286/samyrad2026.339

ABSTRACT
Geometry teaching in South African higher education continues to face challenges related to student engagement, conceptual understanding, and visualising abstract spatial relationships. Traditional exemplification, reliant on static diagrams and chalkboard illustrations, limits opportunities for pre-service teachers to explore geometric concepts dynamically. This paper reports an emerging, work -in-progress study investigating how digital tools, including dynamic geometry software, document-camera visualisers, and 3D visualisation platforms, enhance geometric exemplification in undergraduate mathematics classrooms. Framed by variation theory and a qualitative, interpretivist case study design, the study examines how seven mathematics teacher educators across five South African institutions choose and use examples during technology-mediated instruction. Data are collected through pre- and post-observation interviews and video-recorded classroom observations; the pre-observation phase was completed in semester one, with the classroom observations continuing and post-observation interviews in semester two of 2026. The findings reported here are preliminary and drawn from one of the seven participating teacher educators. Preliminary analysis of classroom video data indicates that this teacher educator paired document-camera visualisers with physical manipulatives to model worked examples, such as calculating the surface area of cylinders and triangular prisms, moving systematically from concrete objects to generalised formulae. This pairing appears in this single case, to support contrast and generalisation, key variation-theory patterns, while raising early considerations around instructor readiness and equitable access that remain to be tested against the full participant sample.

KEYWORDS Geometry; digital technology integration; exemplification; variation theory; South African higher education; conceptual understanding.

How to cite: Kgosi, A. M., & Essien, A. A. (2026). Enhancing geometry exemplification through integration of digital technology tools in South African higher education mathematics classrooms. SAMYRAD 2026 Open-Access Proceedings. Intelligent Nudge. https://doi.org/10.68286/samyrad2026.339

Introduction

Mathematics education stands at a critical juncture where traditional pedagogical approaches must evolve to meet the demands of an increasingly digital society. Higher education institutions increasingly recognise technology as fundamental to preparing pre-service teachers (PSTs) for twenty-first-century classrooms [1]. The National Council of Teachers of Mathematics (NCTM, 2023) describes technology as a catalyst for change and innovation and advocates its strategic integration to enrich mathematical learning [2]. Technology use can shift classroom focus from routine calculation toward strategic thinking, fostering deeper conceptual understanding [1], and recent evidence links effective integration to more positive student attitudes and engagement in STEM [3,4].

However, research says little about how exemplification is supported by technology integration. Exemplification, understood as the discursive act of illustrating a mathematical generality through a particular example, is a commonplace of mathematics teaching that offers insight into teachers' instructional decisions and underlying pedagogical reasoning [5,6,7]. Structured exemplification is critical for helping PSTs grasp the object of learning that underlies broader generalisation. Despite the recognised potential of digital tools, teacher educators (TEs) often lack the pedagogical and technological knowledge to integrate examples purposefully when teaching geometry [8], leaving a gap this study addresses by examining how examples are chosen and used when technology is integrated into geometry instruction. This paper reports the study's design and preliminary observations. As an emerging, work-in-progress account, the findings below are illustrative of a single case within the planned sample of seven teacher educators and are not yet generalisable across it.

Theoretical Framework: Variation Theory

This study is framed by variation theory, rooted in phenomenography, which examines how learners discern the critical aspects of an object of learning [9]. A critical aspect is an essential feature of a concept that learners must experience as variation, against a background of invariance in other features, in order to discern it [10,11]. Effective teaching therefore requires purposeful variation of critical features through patterns such as contrast, separation, generalisation, and fusion. Variation theory also distinguishes between the intended, enacted, and lived object of learning: what a teacher plans to teach, what is actually made available in the lesson, and what learners come to discern [7,11]. Combined with the Mathematics Discourse in Instruction framework's account of exemplification [6] variation theory allows this study to analyse not only which examples teacher educators select when integrating digital tools, but how the pattern of variation across those examples shapes what is made available for PSTs to learn about three-dimensional geometry.

Literature Synthesis

Global adoption of digital technologies in mathematics education continues to grow, with evidence pointing to gains in conceptual understanding and engagement (NCTM, 2023). Participatory digital tools have been shown to stimulate mathematical discussion and improve classroom communication [12], while dynamic and interactive tools support hands-on visualisation of abstract concepts [13]. In geometry specifically, tools such as GeoGebra, augmented reality, graphing calculators, and interactive whiteboards have demonstrated effectiveness across educational levels [8]. However, integration remains uneven: teacher readiness, curriculum alignment, and equitable access continue to constrain the extent to which digital tools translate into enriched exemplification [1,3]. Few studies, however, examine the specific examples teacher educators choose and how these are made available through technology that is the focus taken up by the research questions below.

Research Questions

Main Research Question

What examples are chosen and used when integrating digital technology tools in mathematics instruction to promote deep learning in geometry?

Sub-Research Questions

1a. Which exemplification affordances are provided during the integration of technologies in supporting specific geometric concepts?

1b. How does exemplification support the intended, enacted and lived object of learning during the teaching of three-dimensional geometry when integrating technology-enhanced instruction?

1c. What do teacher educators perceive to be the impact of exemplification on students' conceptual understanding and problem-solving skills.

The RQ1b addresses directly how exemplification links intended, enactment and PST learning, the analytical core of variation theory as applied in this study which receives the extensive treatment in the findings below. Whereas RQ 1c is deliberately framed around teacher educators' perceptions of impact, since the study's current data sources are TEs facing rather than PST-facing (see. Research Methodology and Design and Discussion below).

Research Methodology and Design

The study adopts a qualitative, interpretivist case study design [14] to obtain detailed, contextualised accounts of how mathematics teacher educators use exemplification when integrating technology into geometry instruction. Qualitative case study is well suited to examining the complexity and dynamic character of teaching practice within its natural setting [15].

The research is being conducted at five public universities and universities of technology in South Africa that prepare pre-service mathematics teachers. Institutions and participants were purposively selected based on established technology-teaching infrastructure, accessibility, and location convenience. Seven mathematics teacher educators who use digital technology in their practice form the study sample; access is negotiated through faculty heads of department during the ethical clearance process. This study notes that all seven participants use different digital technology tools as resources like dynamic software (e.g., GeoGebra and document-camera visualisers and others). The case reported below (TE1) uses a document -camera visualiser for a 3D visualisation platforms and PowerPoint rather than a dynamic geometry software.

Three data sources are used: semi-structured pre-observation interviews, video-recorded classroom observations, and semi-structured post-observation interviews. The pre-observation interviews, completed in

semester one of 2026, explored participants planned use of examples and technology. Video-recorded classroom observations capture actual classroom exemplification practice, including teacher educators' talk, gestures, use of physical manipulatives, and use of digital tools such as document-camera visualisers and dynamic geometry software. Post-observation interviews, scheduled for semester two of 2026, will probe teacher educators' reflections on the examples used and their intended versus enacted object of learning during the video-recorded classroom observation lessons. Together these methods allow triangulation between planned, enacted, and reflected-upon exemplification practice.

Data Analysis

Data analysis follows a hybrid deductive-inductive thematic analysis [16]. Deductive codes were derived a priori from variation theory and the Mathematics Discourse in Instruction framework: example type (worked/numerical, physical/manipulative, symbolic), pattern of variation (contrast, generalisation, separation, fusion), and object of learning (intended, enacted, lived). Inductive codes were allowed to emerge from repeated viewing of the video data to capture practices not anticipated by the framework, such as the specific affordances of the classroom document-camera visualiser.

The single-case analysis reported below was triangulated across two data sources for the same teacher educator (TE1): video-recorded classroom observation and a semi-structured post-observation interview, itself video-recorded and transcribed. Both were coded using the same intended/enacted/lived object-of-learning categories, together with codes for exemplification affordances and in-the-moment adjustments, allowing the researcher to move beyond what was visible on screen to the teacher educator's own account of her planning, in-the-moment decisions, and reading of PSTs' understanding.

Coding of this material, alongside the pre-observation interview transcripts, is ongoing across the full data set of seven teacher educators. The findings below represent a first cycle of analysis for a single participant, organised by research question rather than by named theme; a fuller thematic synthesis across all seven cases will follow once classroom observations and post-observation interviews are completed in semester two of 2026.

Ethical Considerations

Video recording of classroom interactions was conducted in accordance with relevant institutional research-ethics requirements. Participants were informed about the purpose of the study and the use of recordings and transcripts. Identifying information was removed from reported excerpts, and pseudonyms or participant labels were used where appropriate.

Findings

This preliminary analysis draws on video-recorded observation footage of one teacher educator's lesson on the surface area of three-dimensional solids, captured via a ceiling-mounted projector and a document-camera visualiser. Across six short video excerpts, TE1 worked through two examples in sequence: the total surface area of a cylinder and the total surface area of a triangular prism with the dimensions originally given as base 5 cm, slant sides 3 cm and 4 cm, height 20 cm. TE1 wrote a general formula before substituting the given values step by step for the cylinder (e.g., SA = $2\pi r^2 + 2\pi rh$) and the triangular prism SA = 2(½bh) + H (l + b +Hyp), considering the hypotenuse side of the right-angled triangle, so that the same underlying structure surface area equals the sum of the areas of all faces, was held invariant while the solid itself, and the particular face-area formulae required, varied. For each solid, she wrote a general formula before substituting the each given value step by step. TE1 defines SA = ½bh as half times base times h which refers only to one part of a right -angle triangle only, then TE further explained the addition of 2 times ($½bh$) which is the area of the triangular area with base length of a triangular face (b) and height altitude of a triangular face. Therefore, TE1 continued to explain the perimeter of a triangular base ($l + b + hyp$), multiplied by the height of the prism (H) or Length of the prism (L). These finding provide the exact formula as written on the visualiser. This matches the variation-theory pattern of contrast, varying the solid, held against generalisation, the constant underlying structure.

The document-camera visualiser was the central technological affordance observed across these excerpts, it allowed the teacher educator's real-time handwriting to be magnified and projected for the whole class, functioning as a dynamic substitute for the chalkboard while preserving worked steps for later review, its specific affordances are detailed under RQ1a, below. In several excerpts, this projected worked example was paired with a physical manipulative - a small cylindrical object held up by the TE1, and in an earlier segment, a fold-and-learn 3D geometric-shapes kit visible on the lecture table linking the concrete solid to its symbolic and numerical representation.

RQ1: What examples are chosen and used? Preliminary evidence drawn from a single case and not yet generalised across the full sample - indicates that teacher educators favour worked numerical examples of real-world solids (cylinders, triangular prisms) rather than purely abstract exemplars, and that these examples are built up systematically from a general formula to a numerical solution. Physical manipulatives using everyday objects and commercial fold-and-learn geometric kits were chosen alongside the technology-mediated example, suggesting a deliberate move from concrete to symbolic representation rather than reliance on digital tools alone. TE1 confirmed in the post observation interview that this combination was planned in advance rather than incidental: she reported using PowerPoint slides, the document camera visualiser, and physical solids together in the same lesson "because I already knew that there are my students who learn better when they see solids or when they touch them", alongside others who "learn from diagrams".

RQ1a: Exemplification affordances of the technologies used. The document-camera visualiser afforded three

things a traditional chalkboard could not as easily provide: (i) whole-class visibility of fine handwritten detail through magnification and projection, (ii) preservation of the full worked sequence on screen for learners to revisit, and (iii) flexibility to combine the projected symbolic work with a physical object held in view of the camera. The post-observation interview corroborates this: TE1 described calling a PST forward mid-lesson to draw and project a supporting sketch of the triangular prism once she noticed some PSTs "could not follow" the worked example and separately described using the camera visualiser to let PSTs project and present their own worked solutions for whole-class critique. This suggests the visualiser's affordance extends beyond teacher-led modelling to peer-mediated exemplification and real-time formative feedback ("we critique… you missed this, this is where you went wrong"). These affordances appear to support the coherence of the example space; the visible link between the concrete object, the general formula, and its numerical instantiation; though this requires confirmation against the fuller dataset, beyond this single teacher educator's self-report and one set of video excerpts.

RQ1b: Intended, enacted, and lived object of learning. This question is addressed most directly by triangulating the video data with TE1's post-observation account, which separates the three constructs clearly. The intended and enacted descriptions below draw on both sources; the lived description draws solely on TE1's self-report, since no direct PST-side data is intended to be collected, the study focus is only on TEs practice.

Intended: TE1 reported planning a multi-representational route to the object of learning, the general procedure for finding the total surface area of a 3-D solid, before the lesson began, deliberately combining slides, the visualiser, and physical solids "because I was aware that there would be students who would learn better from the drawings" and others who would not understand "just from the use of solids or by reading the slides".

Enacted: In the video data, the enacted lesson followed this plan through two contrasting worked examples (cylinder, then triangular prism) that varied the solid while keeping the additive face-area structure invariant. TE1's interview reveals a further, unplanned adjustment: when the originally chosen triangular prism was not right-angled and several PSTs "could not follow" the calculation, she substituted a simplified right-angled example ("let's assume the base is a right-angled triangle of dimensions 3, 4 and 5") and brought in a student-drawn, visualiser-projected diagram. This is a clear instance of the enacted object of learning being actively renegotiated in response to visible non-discernment, with the same technological resources used to close the gap.

Lived: TE1's account of what PSTs appeared to take away goes beyond procedural recall to structural understanding: she reported that PSTs came to understand "why one has to calculate areas of the faces of a 3-D solid and add them up", and could explain, for a cylinder, that "the length of a rectangle becomes a circumference and the width of the rectangle becomes a height, that's why we have 2πr [multiplied by] height" linking the net of the solid to a previously learned area formula. She further reported that PSTs generalised the procedure beyond the taught example ("for any triangular prism to calculate surface area… it depends on the dimensions of the triangle and the height of the prism") and, when subsequently given a task without accompanying drawings, were able to visualise the solid mentally evidence, in TE1's account, of a shift from concrete to abstract representation.

Relationship among the three: Asked directly about the relationship between what she intended, enacted, and what PSTs appeared to experience, TE1 stated that "what happened is everything that I planned to achieve", attributing the alignment to a multi-representational design that had anticipated diverse PST learning preferences in advance. In this case, then, enacted and (by the teacher educator's own report) lived objects of learning tracked closely with what was intended, in part because the one significant contingency that arose mid-lesson was resolved using the same representational and technological resources that had been planned for other purposes. This is a single, teacher-reported case, and PST-side data (from PST interviews or work samples) will be needed to corroborate the lived object of learning independently of the teacher educator's perspective.

RQ1c: Perceived impact on conceptual understanding and problem-solving: This study's data sources are entirely teacher-educator-facing at this stage; this sub-question is addressed here as TE1's *perceived* impact rather than as independently measured PST outcomes; direct evidence of PST problem-solving will require PST-facing data collection in later phases. Full impact cannot yet be claimed given the study's ongoing status and a single case scope. However, converging evidence from the video and interview data points to a consistent early pattern in TE 1's account: the visualiser-supported, step-by-step reconstruction of formulae appeared to make the logical structure of each calculation explicit, and TE1 reported that PSTs performed well on subsequent tasks, with one PST presenting a worked solution on the visualiser for the class to check against their own answers and, where errors occurred, to jointly identify "where you went wrong". This peer-checking routine, combined with TE1's account of PSTs solving a later task without diagrams, suggests, in this teacher educator's perception, that real-time, technology-mediated modelling may support both error-noticing and a degree of conceptual transfer. Confirmatory evidence, particularly direct evidence of PSTs' problem-solving rather than the teacher educator's report of it, will depend on the completed interview and observation dataset across all seven participants and ideally on the addition of a PST-facing data source (see discussion below).

Discussion

Based on variation theory, TE1's case offers a concrete illustration of how contrast and generalisation can be engineered through technology-mediated exemplification: holding the additive face-area structure invariant while varying the solid (cylinder, then triangular prism) is a clear instance of the contrast/generalisation pairing described by variation theory [9,10], and the document-camera visualiser appears, in this case, to

have been the technological resource that made that pairing visible and reviewable for the whole class in close to real time - consistent with the broader literature on participatory and interactive digital tools supporting mathematical communication and hands-on visualisation [12,13]. The mid-lesson substitution of the triangular-prism example is, on this reading, a particularly clear instance of the enacted object of learning being renegotiated in response to visible non-discernment, and it is the strongest evidence in this case for a claim variation theory makes but that is not often documented directly in classroom data: that teachers can and do adjust their pattern of variation in situ when a first attempt does not secure discernment.

At the same time, the case is a single one, and its findings should be read as illustrative rather than representative. Three limits are worth stating explicitly, since they bear directly on how the emerging implications below should be read. First, the Lived object of learning and the perceived impact reported under RQ1c rest entirely on TE1's self-report; no PST interviews, work samples, or assessment data have yet been collected, so what PSTs actually discerned remains inferred rather than observed. Second, TE1's technology use in the analysed excerpts - a document-camera visualiser and PowerPoint is narrower than the dynamic geometry software and 3D visualisation platforms foregrounded in the Introduction and Literature Synthesis; whether or how the six other participants use such tools is not yet established in this report. Third, the equitable-access and infrastructure concerns raised in the literature [1,3] are not directly evidenced in this single case, since TE1's institution evidently had working visualiser and projection infrastructure throughout; any claim about infrastructure access at this stage draws on the wider literature rather than on this study's data.

Emerging Implications

This single case suggested for further inquiry and practice, rather than as generalisable recommendations; they will be revisited once data collection across all seven participating teacher educators is complete.

First, TE1's practice suggests that mathematics teacher education programmes could consider pairing digital visualisation tools (document-camera visualisers, and for participants who use dynamic geometry software) with physical manipulatives to support movement between concrete, pictorial, and symbolic representations of geometric concepts; whether this pattern holds across the wider sample remains to be seen.

Second, the mid-lesson substitution described under RQ1b suggests that professional development for teacher educators could usefully address the design of contrasting example sequences, informed by variation theory, and the in-the-moment adjustment of those sequences when a first example does not secure discernment, rather than treating technology adoption as sufficient in itself - though this, too, is based on one teacher educator's practice and account.

The equitable-access question raised in the wider literature [1,3] whether institutions can reliably provide the projection and visualiser infrastructure this kind of teaching depends on, is an important one, but this study's data do not yet speak to it directly; we flag it here as a question for the completed dataset, rather than as a recommendation this case supports.

Conclusion

This paper has reported the design and preliminary, single - case, observations of an ongoing, emerging study into how South African mathematics teacher educators exemplify geometric concepts through digital technology integration. Early evidence from TE1's classroom video and post-observation interview data suggests that a document-camera visualisers, used alongside physical manipulatives can support a structured, generalisable approach to teaching three-dimensional geometry consistent with variation theory including an instance of the enacted object of learning being renegotiated mid-lesson in response to PST's visible non-discernment. As data collection continues into semester two of 2026 across all seven participating teacher educators, and as PST-facing data are considered for later phases, subsequent analysis will more fully address the affordances, enacted objects of learning, and perceived impact on PSTs' conceptual understanding across the participating institutions, with implications for mathematics teacher preparation and curriculum design in South African higher education to be developed once that fuller picture is available.

References

[1] Atteh, E., Assan-Donkoh, I., Ayiku, F., Nkansah, E., & Adams, A. K. (2020). The Use of Technology among School Mathematics Teachers and Students: The New Wave of Recommended Instructions. Asian Research Journal of Mathematics, 16(5), 18–29. https://doi.org/10.9734/arjom/2020/v16i530189

[2] National Council of Teachers of Mathematics. (2023). Equitable integration of technology for mathematics learning [Position statement]. https://www.nctm.org/standards-and-positions/equitable-integration-of-technology

[3] Hidayat, A., & Firmanti, P. (2024). Navigating the tech frontier: a systematic review of technology integration in mathematics education. Cogent Education, 11(1). https://doi.org/10.1080/2331186X.2024.2373559

[4] Pitchford, N.J., Gulliford, A., Outhwaite, L.A., Davitt, L.J., Katabua, E., Essien, A.A. (2021). Using Interactive Apps to Support Learning of Elementary Maths in Multilingual Contexts: Implications for Practice and Policy Development in a Digital Age. In: Essien, A.A., Msimanga, A. (eds) Multilingual Education Yearbook 2021. Multilingual Education Yearbook. Springer, Cham. https://doi.org/10.1007/978-3-030-72009-4_8

[5] Adler, J., & Davis, Z. (2006). Opening another black box: Researching mathematics for teaching in mathematics teacher education. Journal for Research in Mathematics Education, 37(4), 270–296.

[6] Adler, J., & Ronda, E. (2015). A framework for describing mathematics discourse in instruction and interpreting differences in teaching. African Journal of Research in Mathematics, Science

and Technology Education, 19(3), 237–254. https://doi.org/10.1080/10288457.2015.1089677
[7] Essien, A.A. (2021). Rethinking exemplification in mathematics teacher education multilingual classrooms. In: Kollosche D (ed.), Exploring New Ways to Connect: Proceedings of the eleventh international mathematics education and society conference. Tredition, Austria, 24–29 September, vol. 1. pp. 39–55. https://doi.org/10.5281/zenodo.5457116
[8] Sunzuma, G. (2023). Technology integration in geometry teaching and learning: A systematic review (2010–2022). LUMAT: International Journal on Math, Science and Technology Education, 11(3), 1–18.
[9] Marton, F., & Booth, S. (1997). Learning and awareness. Lawrence Erlbaum Associates.
[10] Marton, F., & Pang, M. F. (2006). On Some Necessary Conditions of Learning. Journal of the Learning Sciences, 15(2), 193–220. https://doi.org/10.1207/s15327809jls1502_2.
[11] Pang, M. F., & Marton, F. (2005). Learning theory as teaching resource: Enhancing students' understanding of economic concepts. Instructional Science, 33(2), 159–191. https://doi.org/10.1007/s11251- 005-2811-0
[12] Calor, S. M., Dekker, R., van Drie, J. P., & Volman, M. L. (2022). Scaffolding small groups at the group level: Improving the scaffolding behaviour of mathematics teachers during mathematical discussions. Journal of the Learning Sciences, 31(3), 369–407.
[13] Attard, C., & Holmes, K. (2020). "It gives you that sense of hope": An exploration of technology use to mediate student engagement with mathematics. Heliyon, 6(1), e02945. https://doi.org/10.1016/j.heliyon.2019.e02945
[14] Creswell, J. W., & Poth, C. N. (2018). Qualitative inquiry and research design: Choosing among five approaches (4th ed.). SAGE.
[15] Yin, R. K. (2018). Case study research and applications: Design and methods (6th ed.). SAGE.

[16] Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. Qualitative Research in Psychology, 3(2), 77–101.